Assignment of elementary school students to high schools

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Bachelor Final Project – Applied Mathematics

Problem in Amsterdam

In Amsterdam there are approximately 70 high schools and every year about 7500 students who finished elementary school. It happens quite a lot that too many students want to go to the same high school. Every school has a maximum capacity of new students they can accept.

Preference list

To solve this problem all the new students in Amsterdam have to make a preference list of high schools. The students may indicate their personal preference by ranking the schools they would like to go to. There are different methods available to assign everyone to a high school with these lists. The methods have their pros and cons. For example, *Pareto efficiency* is a desired property. It means that, after everybody is assigned, no student can find others that would like to switch schools.

New method

In my final project I have worked on a new method that is also Pareto efficient. It is based on finding a *matching* in a *bipartite graph*. A bipartite graph is a drawing with on one side a dot for every school (the red dots), and on the other side dots for all the students (the blue dots). There is a line in this graph between a red and a blue dot if the student has that school on his or her preference list. Finding a matching means that we have to choose one line for each student in order to decide which school he or she will be assigned to. Of course, we have to make sure that we do not leave too many lines attached to the same school. The number of chosen lines per red dot needs to be below (or exactly at) the capacity of that particular school. One disadvantage of the new method is that it works best when all students submit a preference list of the same length.

Weight

We wrote a computer program to find an optimal solution for the problem in Amsterdam, because in reality the graph is really big! By giving the lines a weight we could tell the computer which lines are important (for example, from a student to his or her favorite school) and which lines are less important (for instance, between a student and a school that is quite low on his or her preference list).

Results

With this method we discovered that almost every student could have been placed in their top 3.
In 2015 only six students would be assigned to their fourth preferred school, and nobody lower than that. In reality 417 students were assigned outside their top 3, of which 65 students were placed on a school that was not even on their original preference list!
In 2016 we had only one student who we could not place in his or her top 3 with the new method, while in reality 369 students were assigned to a school outside their top 3.

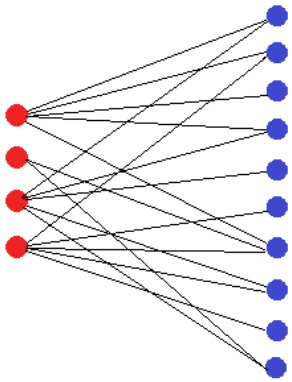

Figure 1: An example of a bipartite graph with schools and students.